DEPARTMENT OF MATHEMATICS UNIVERSITY OF NIJMEGEN The Netherlands

UNDERSTANDING AND USING BROUWER'S CONTINUITY RINCI LE

Wim Veldman

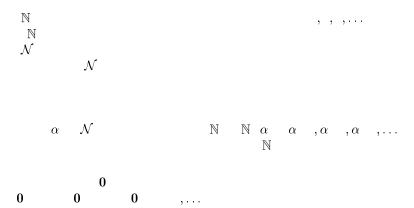
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Understanding and using Brouwer's Continuity Principle

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1 Brouwer's ontinuity Principle



example j < example n example njexample , , ,...

0

 α

 α

 $R \subseteq \mathcal{N} \times \mathbb{N}$ $\langle \alpha, m \rangle$ R αRm m α α \mathcal{N} \mathbb{N} αRm m α \mathcal{N} m \mathbb{N} m α α mm α m β

 m, n, \dots

 \mathbb{N}

$$\begin{split} R &\subseteq \mathcal{N} \times \mathbb{N} \\ \forall \alpha \in \mathcal{N} &\exists m \ \alpha R m \\ \forall \alpha \in \mathcal{N} &\exists n \ \exists m \ \forall \beta \in \mathcal{N} \end{split} \qquad \qquad i < n \ \alpha \ i \qquad \beta \ i \qquad \beta R m \end{split}$$

 $WC - \mathbb{N}$

The continuity of real functions $\mathbf{2}$

2.2
$$X\subseteq\mathcal{N} \qquad R\subseteq X\times\mathbb{N}$$

$$\forall \alpha\in X\;\exists m\;\;\alpha Rm \qquad \qquad i< n\;\;\alpha\;i \quad \beta\;i \qquad \beta Rm$$

$$r\quad\mathcal{N} \qquad X \qquad \qquad \alpha \quad X\;r\;\alpha \quad \alpha$$

2.3
$$q,q,\ldots$$
 \mathbb{Q}
$$n \mid q - q \mid < --- \mathbb{R}$$

$$lpha,eta$$
 γ $lpha$ γ

2.9
$$b, b, \ldots$$
 \mathbb{Q} $i k b \leq \alpha i$ $b \leq \alpha i k$ $\alpha i \leq b i$ $\alpha i k \leq b i$ $\alpha \mathbb{R}$

$$\mathbf{2.10}$$
 Theorem:

 \mathbb{Q}

3 Strong counter-examples

$$\neg \forall \alpha \in \mathcal{N} \quad \alpha \quad \mathbf{0} \lor \neg \quad \alpha \quad \mathbf{0}$$

$$\forall \alpha \in \mathcal{N} \quad \alpha \quad \mathbf{0} \lor / \quad \alpha \quad \mathbf{0}$$

$$\alpha \quad \mathcal{N} \qquad \qquad i < n \quad \alpha \quad i \qquad \qquad \alpha \quad \mathbf{0}$$

$$\alpha \quad \mathcal{N} \qquad \qquad i < n \quad \alpha \quad i \qquad \qquad \neg \quad \alpha \quad \mathbf{0}$$

f n f —

3.3

 $, \qquad \mathbb{R}$

4 Brouwer's first application

4.1 Theorem:

4.2

4.3

 \mathcal{N}

T α $\mathcal N$

$$T$$

$$\mathbf{0},\;\langle\;\rangle*\mathbf{0},\;\langle\;,\;\rangle*\mathbf{0},\;\langle\;,\;,\;\rangle*\mathbf{0},\;\ldots$$
 example
$$T$$

T

$$T$$
 \mathbb{N} T

4.6 Theorem:

4.8 Theorem:

 $n,m \qquad n\cdot T \qquad \qquad n \qquad \qquad \cdot T \qquad \qquad T \qquad \qquad .$

4.9
$$p \cdots \omega \qquad p \qquad \langle n, n, \dots, n \rangle \\ \langle p, p, \dots, p \rangle$$

$$\begin{matrix} k & & & & & & \\ & & \mathcal{N} & & & & \mathbb{T} \ \omega & \cdot p & \omega & \cdot p & \cdots \\ \omega & & \cdot p & & p \cdot T & \oplus p \cdot T & \oplus \dots \oplus p & \cdot T & & < \end{matrix}$$

4.10 Theorem:

$$\alpha, \beta \quad \omega \quad \alpha < \beta \quad \mathbb{T} \quad \alpha \quad \mathbb{T} \quad \beta$$

5 A model-theoretic observation

5.2 Theorem:

$$\alpha,\beta \quad \omega \qquad \alpha < \beta \qquad \mathbb{T} \ \alpha \qquad \qquad \mathbb{T} \ \beta$$

$$\exists x \ \varphi \ x \ \land \forall y \ \varphi \ y \ \rightarrow y \quad x$$

$$\exists x \qquad \qquad x \qquad \qquad T$$

6 Beginning the Borel hierarchy

 Π Σ

X

X

 $\begin{array}{ccc} \mathbf{PosIrr} & & \\ & \mathbf{PosIrr} & \\ & \mathbf{Rat} & \mathbf{\Sigma} & \\ & \mathbf{\Pi} & & \mathbb{R} \end{array}$

6.3 Theorem:

$$\mathbf{Rat} \subseteq G \qquad \qquad \alpha \quad \mathbf{PosIrr} \qquad \qquad \alpha \in G$$

$$G, G, \dots \qquad \qquad \mathbf{Rat} \subseteq G$$

$$\mathbf{Rat} \subseteq G \qquad \qquad \alpha \qquad \qquad n$$

$$q \quad < q \quad < q \qquad < q \qquad \qquad n$$

$$\alpha \quad , \alpha \quad , \dots , \alpha \quad n- \quad , \alpha \quad n- \quad \qquad q \quad < q$$

$$q \quad < q \quad < q \qquad \qquad q \quad < q$$

6.4 Theorem:

6.5

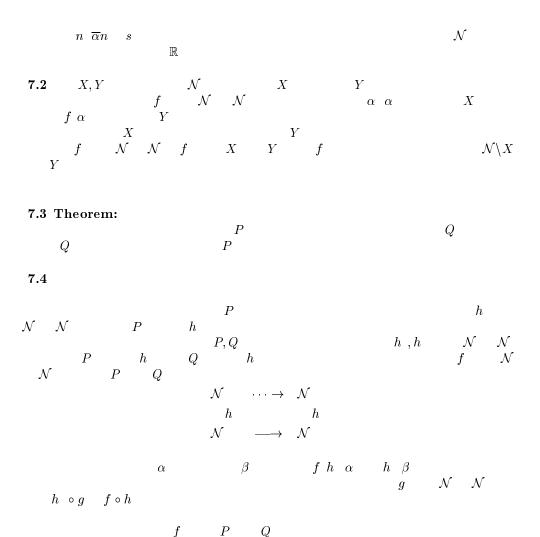
 Σ Π

6.6 Theorem:

6.7

7 Borel hierarchy theorems

7.1 $\mathcal{N} \qquad \qquad X \quad \mathcal{N} \qquad \qquad s \quad \mathbb{N} \qquad \qquad X \qquad \mathcal{N} \qquad \qquad \alpha \quad \mathcal{N} \qquad \qquad \mathcal{N} \qquad \mathcal{N} \qquad \qquad \mathcal{N}$



7.6 Theorem:

$$P$$
 P Finite $\subseteq Q \subseteq \mathbf{Almostfinite}$

Almostfinite

7.7
$$\alpha, \beta$$
 $\alpha \in \beta$ $\alpha \in \beta$

7.8 Theorem:

$$Q$$
 P Share T Almostfinite $Σ$ Share T

References