Restriction on Longitudinal Waves

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Compare this with:

$$x_k = k\,.\,\Delta$$

Then the condition $x_{k+1} > x_k$ translates into:

$$\Delta - A\,.\,sin\left(\frac{2\pi}{\lambda}\Delta\right) > 0 \quad\Longrightarrow\quad A < \frac{\Delta}{sin(2\pi\Delta/\lambda)}$$

We can prove that a sufficient conditon is $A < \lambda/(2\pi)$, because $sin(t)/t < 1$ and hence:

$$A < \frac{\lambda}{2\pi} < \frac{\lambda/(2\pi)}{sin(t)/t} = \frac{\lambda}{2\pi}\frac{2\pi\Delta/\lambda}{sin(2\pi\Delta/\lambda)} = \frac{\Delta}{sin(2\pi\Delta/\lambda)}$$

It is concluded herefrom that: *a sufficient condition for a longitudinal wave to exist is that its amplitude shall be smaller than its wavelength* λ *divided by* 2π. The significance of this condition is that it is *independent* of the size of the increment Δ, which therefore can be chosen arbitrary (small).

In message <1103238510.845881.288470@f14g2000cwb.googlegroups.com> Edward Green writes that I

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seem to have taken a simple idea and converted it into an impenetrable
if impressively typeset line of math! One shudders to think what might
happen if you exposited something truly difficult. ;-)
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There is a much easier way to arrive at the result, indeed. Let the transversal wave be given by:

$$y(t) = A\,.\,sin\left(\frac{2\pi}{\lambda}t\right)$$

Let it be assumed that the longitudinal wave can be arbitrarily dense. Then the possibility of the construction turns out to be equivalent with the requirement that the slope of the tangent line at any point of the transversal wave is greater than minus one:

$$\frac{dy}{dt} \geq -1 \quad\Longrightarrow\quad A\frac{2\pi}{\lambda}\,.\,cos\left(\frac{2\pi}{\lambda}t\right) \geq -1$$

When specified for the points where the cosine function reaches its minimum:

$$\frac{2\pi}{\lambda}t = k.\pi \quad\Longrightarrow\quad \frac{dy}{dt} = A\frac{2\pi}{\lambda}\,.\,(-1) \geq -1 \quad\Longrightarrow\quad A \leq \frac{\lambda}{2\pi}$$

Meaning that we are just done.
Thanx, Edward!

Disclaimers

Anything free comes without referee :-(
My English may be better than your Dutch.