Newtonian gravitational deflection of light revisited

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The angle of deflection of a light ray by the gravitational field of the Sun, at grazing incidence, is calculated by strict and straightforward classical Newtonian means using the corpuscular model of light. The calculation is presented in the historical and scientific contexts of Newton's *Opticks* and of modern views of the problem.

I. INTRODUCTION

Isaac Newton's *Opticks* [2] — whose last edition, the fourth, was published in 1730 — is entirely dedicated to the description of experiments in optics. The subtitle reads *"A Treatise of the Reflections, Refractions, Inflections & Colours of Light"*. It is composed by three Books. Book One synthesizes the previous knowledge on the properties of light and is presented as a series of definitions, axioms and propositions. Books Two and Three are devoted to Newton's experiments and to the many discoveries he made therefrom. Book Three ends with a series of 31 queries. In Newton's words: *"When I made the foregoing Observations, I design'd to repeat most of them with more care and exactness, (...) But I was then interrupted, and cannot now think of taking these things into farther Consideration. (...) I shall conclude with proposing only some Queries, in order to a farther search to be made by others."* [3].

The modern concept of the gravitational deflection of light is very much an Einsteinian idea [4, 5], though traces of the idea can be found in Newton's Query 1, namely, *"Do not Bodies act upon Light at a distance, and by their action bend its Rays, and is not this action (cæteris paribus) strongest at the least distance?"*. In conjunction with its corpuscular model of light, which is explicitly stated in Query 29 as *"Are not the Rays of Light very small Bodies emitted from shining Substances?"*, one has the whole scenario for a Newtonian calculation of the gravitational deflection of light by a massive body.

However there is not such a discussion in the *"Opticks"*. One should remember that Newton was entirely concerned with the properties of light behavior in his daily laboratory experiments.

In the next section I shall pursue on such a scenario and calculate the Newtonian deflection of a light ray at grazing incidence in the solar limb. The result is precisely the same as obtained with modern space-time curvature calculations except for a factor of two. For exam-

ple, Weinberg [6] obtains the general solution for grazing incidence at the Sun. The solution holds for various theories of gravity including General Relativity and Newton's gravity. Denoting the angles of deflection for these cases by δ_{GR} and δ_{NG}, Weinberg's general expression yields

$$\delta_{GR} = \frac{4GM_S}{c^2 R_S} = 1.75 \text{ arcsec} \qquad (1)$$

and

$$\delta_{NG} = \frac{2GM_S}{c^2 R_S} = \frac{1}{2}\delta_{GR}, \qquad (2)$$

where G, c, M_S and R_S are the gravitational constant, the speed of light, the Sun's mass and the Sun's radius, respectively.

Einstein's first calculation of the gravitational deflection of light, in 1911 (see [4], for a historical account and scientific references), was performed using the Equivalence Principle and the equivalent mass-energy of a photon. The calculation yielded δ_{NG}. Only in his second calculation, published in 1916, where he included the effect of space-time curvature, he obtained a value twice as large as his first calculation, i.e., δ_{GR} [4].

Before going on with the classical derivation of δ_{NG}, it is worthwhile to mention an interesting note by I. Bernard Cohen, in the Preface of the modern edition of *Opticks* used here. On page xxxiii, he comments about the Queries: *"To be sure, the speculations of the Opticks were not hypotheses, at least to the extent that they were framed in questions. Yet if we use Newton's own definition, that 'whatever is not deduced from the phenomena is to be called an hypothesis' they are hypotheses indeed. The question form may have been adopted in order to allay criticism, but it does not hide the extent of Newton's belief. For every one of the Queries is phrased in the negative! Thus Newton does not ask in truly interrogatory way (Qu. 1) : 'Do Bodies act upon Light at a distance... ?' — as if he did not know the answer. Rather, he puts it: 'Do not Bodies act upon Light at a distance... ?' — as if he knew the answer well — 'Why, of course they do!' "*

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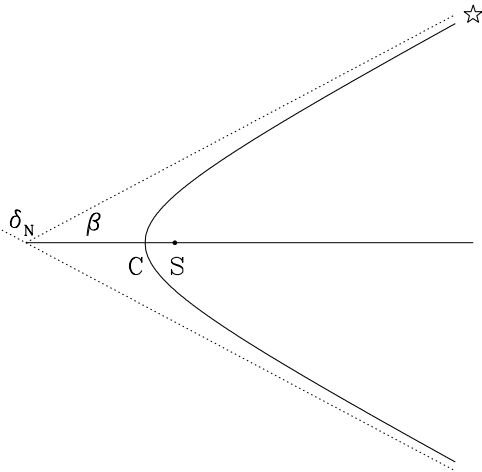URL: http://www.fisica.ufmg.br/~dsoares

FIG. 1: The light ray from a star follows an unbound hyperbolic orbit about the Sun. For deflection on grazing incidence, the distance CS is R$_S$, the solar radius. For illustration purposes, the bending is greatly exaggerated.

II. DEFLECTION OF A LIGHT RAY BY THE SUN

Taking Queries 1 and 29 above at their face values, one may proceed to calculate the deflection of a light ray by the gravitational field of a massive body.

A light ray, from a distant star, under the Sun's gravitational force field describes the usual central force hyperbolic orbit. The deflection of the light ray is illustrated in Fig. 1, with the bending greatly exaggerated for a better view of the angle of deflection.

At grazing incidence through the solar limb, the distance CS is the solar radius R_S. The angle β of the asymptote to the hyperbole of eccentricity ϵ is given by [7]

$$\cos\beta = \frac{1}{\epsilon}. \tag{3}$$

The angle of deflection of the light ray, δ_N, is shown in the figure and is

$$\delta_N = \pi - 2\beta. \tag{4}$$

For a light particle with mass m, total energy E and angular momentum L, with respect to the Sun's center S, the eccentricity is

$$\epsilon = \left(1 + \frac{2EL^2}{G^2m^3M_S^2}\right)^{1/2}. \tag{5}$$

The constants of motion E and L are easily evaluated at point C. Assuming that the particle speed at that point is v one has

$$E = \frac{1}{2}mv^2 - \frac{GmM_S}{R_S} \tag{6}$$

and

$$L = mvR_S. \tag{7}$$

Since the light particle speed is very large — much larger than the escape speed at the solar radius — one can neglect the change in its magnitude and assume that the gravitational field changes only the velocity direction. Making $v \equiv c$, the light speed in vacuum, the eccentricity is written as

$$\epsilon = \left[1 + \frac{\left(c^2 - 2GM_S/R_S\right)c^2R_S^2}{G^2M_S^2}\right]^{1/2}. \tag{8}$$

As expected, the light particle mass m cancels out above. Also, the second term inside the parentheses is readily verified to be much smaller than c^2. Thus, the eccentricity may be simplified to

$$\epsilon = \frac{c^2R_S}{GM_S} = 4.7 \times 10^5 \gg 1. \tag{9}$$

The angle of deflection, given by eq. 4, can be written, with the aid of eqs. 3 and 9 as

$$\delta_N = \pi - 2\cos^{-1}\left(\frac{GM_S}{c^2R_S}\right) \tag{10}$$

and since $x \equiv GM_S/c^2R_S \ll 1$, one can expand $\cos^{-1}x$ in a Taylor's series:

$$\cos^{-1}x = \frac{\pi}{2} - \sin^{-1}x = \frac{\pi}{2} - \left(x + \frac{1}{2}\frac{x^3}{3} + ...\right). \tag{11}$$

Considering only the first term inside the parentheses, δ_N in eq. 10 takes the form of Weinberg's Newtonian deflection given by eq. 2:

$$\delta_N = \frac{2GM_S}{c^2R_S}. \tag{12}$$

In conclusion, there is still the question of why Newton did not discuss the possibility of light ray deflection by a massive heavenly body. Of course, he was well acquainted with the relevant astronomical observations. Solar eclipses were certainly of his knowledge and could certainly motivate digressions on the gravitational bending of light. Nevertheless, there is nothing about such

issues in the *Opticks*. As mentioned above, his main concerns were with the laboratory and daily-life behavior of light. Incidentally, North [8] states that the *"likelihood of such an effect had previously been maintained by Newton and Laplace..."* but gives no references. To my knowledge, the first Newtonian approach to the problem of the gravitational bending of light was undertaken by Johann G. von Soldner, in early XIX century. Soldner's pioneering work is brilliantly reviewed by Stanley L. Jaki [1], who provided also an English translation of his article, submitted for publication in 1801 and printed in 1804.

[1] S.L. Jaki, *Johann Georg von Soldner and the Gravitational Bending of Light, with an English Translation of His Essay on It Published in 1801*, 1978, Foundations of Physics, **8**, pp. 927-950

[2] I. Newton, *Opticks*, (Dover, New York, 1979).

[3] Ibid., pp. 338-339.

[4] D. Overbye, *Einstein in Love: A Scientific Romance*, (Viking Press, New York, 2000), chapters 14 and 25.

[5] A. Pais, *Subtle Is the Lord: The Science and the Life of Albert Einstein*, (Oxford University Press, Oxford, 1983), chapter 11.

[6] S. Weinberg, *Gravitation and Cosmology*, (Wiley, New York, 1972), p. 188.

[7] K. Symon, *Mechanics*, (Addison-Wesley, Reading, 1967), p. 130.

[8] J.D. North, *The Measure of the Universe: A History of Modern Cosmology*, (Dover, New York 1990), p. 68.