Fuzzyfied Sine function

The fuzzyfication of the sine function is defined by:

$$\overline{\sin}(\omega t) := \int_{-\infty}^{+\infty} \Im(e^{i\omega\xi})\, \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}(t-\xi)^2/\sigma^2}\, d\xi$$

Which is the $\Im$maginary part of:

$$\int_{-\infty}^{+\infty} e^{i\omega\xi}\, \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}(t-\xi)^2/\sigma^2}\, d\xi$$

$$= \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}(t-\xi)^2/\sigma^2 + i\omega\xi}\, d\xi$$

Where:

$$-\frac{1}{2}(t-\xi)^2/\sigma^2 + i\omega\xi = -\frac{1}{2}\left[t^2 - 2t\xi + \xi^2 - 2i\omega\xi\sigma^2\right]/\sigma^2$$

$$= -\frac{1}{2}\left[\xi^2 - 2\xi\left\{t + i\omega\sigma^2\right\} + \left\{t + i\omega\sigma^2\right\}^2 - \left\{t + i\omega\sigma^2\right\}^2 + t^2\right]/\sigma^2$$

$$= -\frac{1}{2}\left[\xi - \left\{t + i\omega\sigma^2\right\}\right]^2/\sigma^2 + \frac{1}{2}\left[\left\{t + i\omega\sigma^2\right\}^2 - t^2\right]/\sigma^2$$

$$= -\frac{1}{2}\left[\xi - t - i\omega\sigma^2\right]^2/\sigma^2 + i\omega t - \frac{1}{2}\omega^2\sigma^2$$

Giving:

$$\overline{\sin}(\omega t) = \Im\left(\int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}}\, e^{-\frac{1}{2}\left[\xi - t - i\omega\sigma^2\right]^2/\sigma^2}\, d\xi \;.\; e^{+i\omega t - \frac{1}{2}\omega^2\sigma^2}\right) \quad\Longrightarrow$$

$$\overline{\sin}(\omega t) = e^{-\frac{1}{2}\omega^2\sigma^2}\, \sin(\omega t)$$

Thus the fuzzyfication of a sine is again a sine, but with a smaller amplitude. The amplitude strongly depends upon the (angular) frequency of the sine and the spread of the Gaussian in use.