Fuzzyfied Riemann function

The fuzzyfication of the (real valued) Riemann function is:

$$\overline{f}(x) := \int_{-\infty}^{+\infty} f(\xi)\, \frac{1}{\sigma\sqrt{2\pi}}\; e^{-\frac{1}{2}(x-\xi)^2/\sigma^2}\; d\xi \qquad \text{where:} \qquad f(\xi) = \sum_{k=1}^{\infty} \frac{1}{k^\xi}$$

Working out:

$$\overline{f}(x) = \sum_{k=1}^{\infty} \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}}\; e^{-\xi\ln(k)}\; e^{-\frac{1}{2}(x-\xi)^2/\sigma^2}\; d\xi =$$

$$\sum_{k=1}^{\infty} \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}}\; e^{-\frac{1}{2}(x-\xi)^2/\sigma^2 - \xi\ln(k)}\; d\xi$$

Where:

$$-\frac{1}{2}(x-\xi)^2/\sigma^2 - \xi\ln(k) = -\frac{1}{2}\left[x^2 - 2x\xi + \xi^2 + 2\xi\ln(k)\sigma^2\right]/\sigma^2$$

$$= -\frac{1}{2}\left[\xi^2 - 2\xi\left\{x - \ln(k)\sigma^2\right\} + \left\{x - \ln(k)\sigma^2\right\}^2 - \left\{x - \ln(k)\sigma^2\right\}^2 + x^2\right]/\sigma^2$$

$$= -\frac{1}{2}\left[\xi - \left\{x - \ln(k)\sigma^2\right\}\right]^2/\sigma^2 + \frac{1}{2}\left[\left\{x - \ln(k)\sigma^2\right\}^2 - x^2\right]/\sigma^2$$

$$= -\frac{1}{2}\left[\xi - x + \ln(k)\sigma^2\right]^2/\sigma^2 - x\ln(k) + \frac{1}{2}\ln(k)^2\sigma^2$$

Giving:

$$\overline{f}(x) = \sum_{k=1}^{\infty} \int_{-\infty}^{+\infty} \frac{1}{\sigma\sqrt{2\pi}}\; e^{-\frac{1}{2}\left[\xi - x + \ln(k)\sigma^2\right]^2/\sigma^2}\; d\xi \;.\; e^{-x\ln(k) + \frac{1}{2}\ln(k)^2\sigma^2} \qquad \Longrightarrow$$

$$\overline{f}(x) = \sum_{k=1}^{\infty} \frac{1}{k^x} e^{\frac{1}{2}\ln(k)^2\sigma^2}$$